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Tugas 3 Matematika 2

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1). Selesaikan integral berikut!

a. $\int \sqrt[3]{\cos x} \cdot \sin x \, dx$

Misal : $u = \sqrt[3]{\cos x}$
$= (\cos x)^{1/3}$

$du = \frac{1}{3\sqrt[3]{\cos^2 x}} \cdot -\sin x \, dx \longrightarrow dx = \frac{3\sqrt[3]{\cos^2 x}}{-\sin x} du$

$\longrightarrow \int u \cdot \cancel{\sin x} \cdot \frac{3\sqrt[3]{\cos^2 x}}{-\cancel{\sin x}} du$

$= -3 \int u \cdot u^2 \, du$

$= -3 \int u^3 \, du$

$= -3 \cdot \frac{1}{4} \cdot u^4 + C$

$= -\frac{3}{4} \cos x \cdot \sqrt[3]{\cos x} + C$

b. $\int \sqrt{\tan x} \cdot \sec^4 x \, dx$

misal : $u = \sqrt{\tan x}$

$du = \frac{1}{2\sqrt{\tan x}} \cdot \sec^2 x \, dx \longrightarrow dx = \frac{2 \cdot \sqrt{\tan x}}{\sec^2 x} du$

$\longrightarrow \int u \cdot \cancel{\sec^4 x}^{\sec^2 x} \cdot \frac{2\sqrt{\tan x}}{\cancel{\sec^2 x}} du = 2\int u \cdot u \cdot (1 + u^4) \, du$

$= 2\int u^2 + u^6 \, du$

$= 2\left[\frac{1}{3}u^3 + \frac{1}{7}u^7\right] + C$

$= \frac{2}{3} \tan x \sqrt{\tan x} + \frac{2}{7} \tan^3 x \cdot \sqrt{\tan x} + C$

2). a. Tunjukkan $\int \operatorname{cosec} \theta \, d\theta = -\ln |\operatorname{cosec} \theta + \cot \theta| + C$

$\longrightarrow \int \operatorname{cosec} \theta \, d\theta = \int \operatorname{cosec} \theta \cdot \frac{(\operatorname{cosec} \theta + \cot \theta)}{(\operatorname{cosec} \theta + \cot \theta)} d\theta$

misal : $u = \operatorname{cosec} \theta + \cot \theta$

$du = -\left[\operatorname{cosec} \theta \cot \theta + \operatorname{cosec}^2 \theta\right] d\theta$

$d\theta = \frac{-1}{\operatorname{cosec} \theta \cot \theta + \operatorname{cosec}^2 \theta} du$

$$\rightarrow \int \frac{\cancel{\operatorname{Cosec}^2\theta + \operatorname{Cosec}\theta \operatorname{Cot}\theta}}{u} \cdot \frac{-du}{\cancel{\operatorname{Cosec}^2\theta + \operatorname{Cosec}\theta \operatorname{Cot}\theta}}$$

$$= \int -\frac{1}{u}\, du$$

$$= -\ln u$$

$$= -\ln|\csc\theta + \operatorname{Cot}\theta| + C$$

b. Dari a dapat ditulis $\int \operatorname{cosec}\theta\, d\theta = \ln|\operatorname{cosec}\theta - \operatorname{Cot}\theta| + C$ dan $\int \operatorname{Cosec}\theta\, d\theta = \ln|\tan\frac{1}{2}\theta| + C$

$\bullet\!> \int \operatorname{Cosec}\theta\, d\theta = \ln|\operatorname{Cosec}\theta - \operatorname{Cot}\theta|$

$$\longrightarrow \int \operatorname{Cosec}\theta\, d\theta = \int \operatorname{Cosec}\theta \cdot \frac{(\operatorname{Cosec}\theta - \operatorname{Cot}\theta)}{(\operatorname{cosec}\theta - \operatorname{Cot}\theta)}\, d\theta$$

Misal : $u = \operatorname{Cosec}\theta - \operatorname{Cot}\theta$

$du = -[\operatorname{Cosec}\theta \cot\theta - \operatorname{Cosec}^2\theta]\, d\theta \longrightarrow d\theta = \dfrac{-du}{[\operatorname{Cosec}\theta \cot\theta - \operatorname{Cosec}^2\theta]}$

$$\hookrightarrow \int \frac{\overset{-1}{\cancel{\operatorname{Cosec}^2\theta - \operatorname{Cosec}\theta \operatorname{Cot}\theta}}}{u} \cdot \frac{-du}{\cancel{\operatorname{Cosec}\theta \operatorname{Cot}\theta - \operatorname{Cosec}^2\theta}}$$

$$= \int \frac{1}{u}\, du$$

$$= \ln u + C$$

$$= \ln|\operatorname{Cosec}\theta - \operatorname{Cot}\theta| + C$$

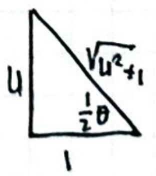
$\bullet\!> \int \operatorname{Cosec}\theta\, d\theta = \ln|\tan\frac{1}{2}\theta| + C$

Misal : $u = \tan\frac{1}{2}\theta$

$\tan^{-1} u = \frac{1}{2}\theta$

$2\tan^{-1} u = \theta$

$du \dfrac{2}{1+u^2} = d\theta$



$\operatorname{Sin}\frac{1}{2}\theta = \dfrac{u}{\sqrt{u^2+1}}$

$\operatorname{Cos}\frac{1}{2}\theta = \dfrac{1}{\sqrt{u^2+1}}$

$$\operatorname{Sin}\theta = 2 .\operatorname{Sin}\tfrac{1}{2}\theta \operatorname{Cos}\tfrac{1}{2}\theta$$

$$= 2 . \frac{u}{\sqrt{u^2+1}} \cdot \frac{1}{\sqrt{u^2+1}}$$

$$= \frac{2u}{u^2+1}$$

$$\rightarrow \int \operatorname{cosec}\theta\, d\theta = \int \frac{1}{\operatorname{Sin}\theta}\, d\theta$$

$$= \int \frac{\cancel{u^2+1}}{2u} \cdot \frac{2}{\cancel{1+u^2}}\, du$$

$$= \int \frac{1}{u}\, du$$

$$= \ln u + C$$

$$= \ln|\tan\tfrac{1}{2}\theta| + C$$

3). Selesaikan :

a. $\int \frac{2t^2+3t+3}{(t+2)^3}\,dt \longrightarrow \int \frac{A}{(t+2)} + \frac{B}{(t+2)^2} + \frac{C}{(t+2)^3}$

$$\frac{2t^2+3t+3}{(t+2)^3} = \frac{A}{(t+2)} + \frac{B}{(t+2)^2} + \frac{C}{(t+2)^3}$$

$$2t^2+3t+3 = A(t^2+4t+4) + B(t+2) + C$$
$$= At^2 + 4At + 4A + Bt + 2B + C$$

- $A = 2$
- $4A + B = 3 \longrightarrow B = -5$
- $4A + 2B + C = 3 \longrightarrow 8 + (-10) + C = 3$
  $C = 5$

$$\hookrightarrow \int \frac{2}{(t+2)} + \frac{(-5)}{(t+2)^2} + \frac{5}{(t+2)^3}\,dt$$

$$= 2\ln|t+2| + \frac{5}{t+2} - \frac{5}{2(t+2)^2} + C$$

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b. $\int \frac{dx}{x^4-3x^3-7x^2+27x-18} \longrightarrow$

$$\begin{array}{c|ccccc} & 1 & -3 & -7 & 27 & -18 \\ 1 & & 1 & -2 & -9 & 18 \\ \hline & 1 & -2 & -9 & 18 & 0 \end{array}$$

$\hookrightarrow x^3 - 2x^2 - 9x + 18$

$$\begin{array}{c|cccc} & 1 & -2 & -9 & 18 \\ 2 & & 2 & 0 & -18 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$\hookrightarrow x^2 - 9$

$$x^4-3x^3-7x^2+27x-18 = (x-1)(x-2)(x+3)(x-3)$$

$$\hookrightarrow \int \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)} + \frac{D}{(x-3)}\,dx$$

$$\hookrightarrow \frac{1}{x^4-3x^3-7x^2+27x-18} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)} + \frac{D}{(x-3)}$$

$$= 1 = A(x^3-2x^2-9x+18) + B(x^3-x^2-9x+9) + C(x^3-6x^2+11x-6) + D(x^3-7x+6)$$

$\cdot\!>$ Substitusi $x = 1$
$-8A = 1$
$A = \frac{1}{8}$

$\cdot\!>$ Substitusi $x = 2$
$-5B = 1$
$B = -\frac{1}{5}$

$\cdot\!>$ Substitusi $x = -3$
$-120C = 1$
$C = -\frac{1}{120}$

$\cdot\!>$ Substitusi $x = 3$
$12D = 1$
$D = \frac{1}{12}$

$$\rightarrow \int \frac{1/8}{(x-1)} + \frac{(-1/5)}{(x-2)} - \frac{1/120}{(x+3)} + \frac{1/12}{(x-3)}\,dx$$

$$= \frac{1}{8}\ln|x-1| - \frac{1}{5}\ln|x-2| - \frac{1}{120}\ln|x+3|$$
$$+ \frac{1}{12}\ln|x-3| + C$$

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4). Selesaikan!

a. $\int e^t \sqrt{1-e^{2t}}\, dt$

misal : $u = e^t$

$du = e^t\, dt$

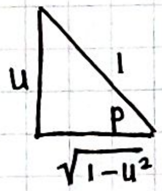
$\int \cancel{e^t} \sqrt{1-u^2} \cdot \frac{du}{\cancel{e^t}}$

$= \int \sqrt{1-u^2}\, du$

→ misal : $u = \sin p$

$du = \cos p\, dp$

→ $p = \sin^{-1} u$



$\int \sqrt{1-\sin^2 p} \cdot \cos p\, dp$

* $\cos 2p = 2\cos^2 p - 1$

$\frac{\cos 2p + 1}{2} = \cos^2 p$

$= \int \sqrt{\cos^2 p} \cdot \cos p\, dp$

$= \int \cos^2 p\, dp$

$= \int \frac{\cos 2p + 1}{2}\, dp$

$= \int \frac{\cos 2p}{2} + \int \frac{1}{2}\, dp$

$= \frac{1}{2} \cdot \frac{1}{2} \sin 2p + \frac{1}{2} p + C$

$= \frac{1}{4} \cdot 2 \sin p \cos p + \frac{1}{2} \sin^{-1} u + C$

$= \frac{1}{2} \cdot \sin(\sin^{-1} u) \cos(\sin^{-1} u) + \frac{1}{2} \sin^{-1} u + C$

$= \frac{1}{2} \left[u \left(\sqrt{1-u^2} \right) + \sin^{-1} u \right] + C$

$= \frac{1}{2} \left[e^t \left(\sqrt{1-e^{2t}} \right) + \sin^{-1}(e^t) \right] + C$ //

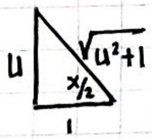
b). $\int \frac{dx}{1 + \sin x - 3\cos x}$

misal : $u = \tan \frac{1}{2} x$

$\frac{x}{2} = \tan^{-1} u$

$x = 2\tan^{-1} u$

$dx = \frac{2}{1+u^2}\, du$



$\sin \frac{x}{2} = \frac{u}{\sqrt{u^2+1}}$

$\cos \frac{x}{2} = \frac{1}{\sqrt{u^2+1}}$

$\sin x = 2 \cdot \frac{u}{\sqrt{u^2+1}} \cdot \frac{1}{\sqrt{u^2+1}}$

$= \frac{2u}{u^2+1}$

$\cos x = \frac{1}{u^2+1} - \frac{u^2}{u^2+1}$

$= \frac{1-u^2}{1+u^2}$

$\int \frac{2\, du}{1 + \frac{2u}{1+u^2} - 3\left(\frac{1-u^2}{1+u^2}\right)} \cdot \frac{1}{1+u^2}$

$= 2. \int \frac{1+u^2}{4u^2+2u-2} \cdot \frac{1}{1+u^2}\, du$

$= 2. \int \frac{\cancel{1+u^2}}{2(2u^2+u-1)} \cdot \frac{1}{\cancel{1+u^2}}\, du$

$= \int \frac{1}{2u^2+u-1}\, du$

→ $\frac{1}{2u^2+u-1} = \frac{A}{(2u-1)} + \frac{B}{(u+1)}$

$1 = Au + A + 2Bu - B$

- $A + 2B = 0$
- $A - B = 1$ −

$3B = -1$

$B = -1/3$

$A - B = 1$

$A = 1 - 1/3$

$= 2/3$

→ $\int \frac{2/3}{(2u-1)}\, du + \int \frac{-1/3}{(u+1)}\, du$

$= \frac{2}{3} \cdot \frac{1}{2} \cdot \ln|2u-1| - \frac{1}{3} \ln|u+1| + C$

$= \frac{1}{3} \ln|2u-1| - \frac{1}{3} \ln|u+1| + C$

$= \frac{1}{3} \ln\left|2\tan \frac{1}{2}x - 1\right| - \frac{1}{3} \ln\left|\tan \frac{1}{2}x + 1\right| + C$

$= \frac{1}{3} \left[\ln \left| \frac{2\tan \frac{1}{2}x - 1}{\tan \frac{1}{2}x + 1} \right| \right] + C$ //

5). Gunakan Substitusi $u = \tan(x/2)$, tunjukkan bahwa :

$$\int \csc x \, dx = \frac{1}{2} \ln \left[\frac{1 - \cot x}{1 + \cot x} \right] + C$$

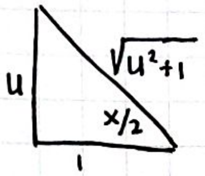
Jawab :

misal : $u = \tan \frac{x}{2}$

$\tan^{-1} u = \frac{x}{2}$

$2 \tan^{-1} u = x$

$du . \frac{2}{1+u^2} = dx$



$\sin x = 2 \sin \frac{1}{2}x . \cos \frac{1}{2} x$

$= 2 . \frac{u}{\sqrt{u^2+1}} . \frac{1}{\sqrt{u^2+1}}$

$= \frac{2u}{u^2+1}$

$\cos x = \cos^2 \frac{1}{2}x - \sin^2 \frac{1}{2}x$

$= \frac{1}{u^2+1} - \frac{u^2}{u^2+1}$

$= \frac{1-u^2}{u^2+1}$

$\longrightarrow \int \csc x dx = \int \frac{1}{\sin x} dx$

$= \int \frac{\cancel{u^2+1}}{\cancel{2}u} du . \frac{\cancel{2}}{\cancel{1+u^2}}$

$= \int \frac{du}{u}$

$= \ln |u| + C$

$= \ln \left| \tan \frac{x}{2} \right| + C$

$= \ln \left| \frac{\sin x/2}{\cos x/2} \right| + C$

$= \ln \left| \frac{\sqrt{\frac{1-\cos x}{\cancel{2}}}}{\sqrt{\frac{1+\cos x}{\cancel{2}}}} \right| + C$

$= \ln \left| \frac{(1-\cos x)^{1/2}}{(1+\cos x)^{1/2}} \right| + C$

$= \ln \left| \left[\frac{1-\cos x}{1+\cos x} \right]^{1/2} \right| + C$

$= \frac{1}{2} \ln \left| \frac{1-\cos x}{1+\cos x} \right| + C$

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※ $\cos x = 2 \cos^2 \frac{1}{2} x - 1$

$\cos^2 \frac{1}{2} x = \frac{\cos x + 1}{2}$

$\cos \frac{1}{2} x = \sqrt{\frac{\cos x + 1}{2}}$

※ $x = 1 - 2 \sin^2 \frac{1}{2} x$

$\sin^2 \frac{1}{2} x = \frac{1 - \cos x}{2}$

$\sin \frac{1}{2} x = \sqrt{\frac{1-\cos x}{2}}$